Characterizing Short Necklace States in Logarithmic Transmission Spectrum of Strongly Localized Systems

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High transmission plateaus exist widely in the logarithmic transmission spectra of localized systems. Their physical origins are short chains of coupled-localized-states embedded inside the localized system, which are dubbed as "short necklace states". In this work, we define the essential quantities and then, based on these quantities, we investigate the short necklace states' properties *statistically* and *quantitatively*. Two different approaches are utilized and the results from them agree with each other very well. In the first approach, the typical plateau-width and the typical order of short necklace states are obtained from the correlation function of *logarithmic* transmission. In the second approach, we investigate statistical distributions of the peak/plateau-width measured in *logarithmic* transmission spectra. A novel distribution is found, which can be exactly fitted by the summation of two Gaussian distributions. These two distributions are the results of sharp peaks of localized states and the high plateaus of short necklace states. The center of the second distribution also tells us the typical plateau-width of short necklace states. With increasing the system length, the scaling property of typical plateau-width is very special since it almost does not decrease. The methods and the quantities defined in this work can be widely used on Anderson localization studies.

PACS numbers: 42.25.Dd, 72.15.Rn, 78.67.Pt, 42.25.Bs

I. INTRODUCTION

The most extraordinary phenomenon of wave transport in random media is Anderson localization[1], which is shared by various systems, such as, electronic, photonic and accoustic systems[2]. In Anderson localized system the ensemble average of logarithmic transmission $\langle lnT \rangle$, not transmission $\langle T \rangle$, is additive with system length L[3,4]. Naturally, the localization length can be defined as $\xi = -2L/\langle lnT \rangle$. In strongly localized systems($L \gg \xi$), the transmission is generally small, but also shows large fluctuations[4]. Those extreme large T values will dominate the transmission of localized systems. In the past years, the study of the physical origins of those large T values yielded abundant results for understanding the transport phenomena in random systems and the Anderson localization[5–25].

Previously, it was pointed out by M. A. Azbel et al.[5] that the resonant transport through those localized states lying near the system center can contribute very large T values, being of order unity. Although such a mechanism can generate high resonant transmission peak[5,6], its contribution to the transmission is vanishingly small in strongly localized system because the resonant peak is exponentially sharp ($\sim e^{-L/\xi}$)[5,6,9]. Laterly, Pendry[7] and Tartakovskii[8] independently predicted a kind of quasi-extended state, called the *necklace state(NS)*. The NS is formed through the coupling between nearly-degenerated localized states which are evenly distributed in the system. Despite the spatial overlaps between localized states are small, *because of the degenerate coupling*, the NS contributes very wide transmission "mini-bands", which can dominate the transmission of strongly localized systems. The NSs have been demonstrated experimentally in photonic systems[10,11]. Their fundamental characters and statistical consequences to the transmission are widely studied[12,14–17]. Recently, it is demonstrated that the NSs have evident contribution to the short-time transport of wave package[19–22] and the dynamics of fluctuations of localized waves[23]. More profoundly, the number of NS can increase dramatically as approaching the Anderson transition point, which strongly supports a modes-coupling induced quantum percolation scenario for the Anderson localization-delocalization transition[24,25].

Even though NSs can contribute very large transmission, their formation has rigorous conditions, i.e., the nearly-degenerated localized states are required to be *evenly distributed* inside the system[7,14,16,17]. In general the localized states with similar frequency are not such well distributed in a specific random configuration. In such cases, *ideal NSs*[14,15] crossing the whole configuration are not formed. But eventually, those nearly-degenerated states which are also *spatially close to each other* couple together, forming short chains of coupled-localized-states embedded inside the configuration. Since those coupled-localized-states have similar properties as the NS, we call them *short necklace state*(SNS)[26]. The SNS also manifests as high transmission plateau which has significant transmission contribution. In transmission spectra the SNS is hardly to be distinguished from isolated localized states since the valley between coupled peaks seems very low. But it can be clearly identified from the logarithmic transmission spectra, for instance, see the coupled-peaks in Fig.1. Because of the strong coupling between the neighboring localized states inside the SNS, on top of the plateau there are sharp peaks and valleys, which correspond to un-smooth changes of the transmission phase between the coupled-peaks[14].

In contrast to the NSs, the SNSs have special properties[26] that (1) the SNSs exist widely in every random configuration while the NS only occurs once in millions of random configurations with large length($L \gg \xi$); (2) the plateau-width of the SNSs only depends on the coupling strength between the neighboring localized states and is insensitive to the system length. With these properties, SNSs are superior *qualitatively* compared with the NSs. However, how to *quantitatively* characterize statistical properties and scaling behaviors of SNSs are still unexplored topic. Even more, some essential quantities of SNS study, such as the "half-width" of SNS plateaus in lnT spectra, need to be defined since there is no obvious physical quantity in previous studies can directly describe SNS.

In this paper, we study the statistical manifestation of the SNS using physical quantities which are defined in the *logarithmic* transmission spectra. To characterize SNS, we find out two different approaches, whose results can be compared with each other. The first approach is based on the correlation functions C_{lnT} (defined in the lnT spectra) and C_t (defined in the transmission coefficient t spectra). From C_{lnT}, we show that there is a typical frequency correlation range of logarithmic transmission, which is explained as the typical SNS plateau-width. From this frequency range and compared with the results of C_t, we can find the most-probable order of SNS. The second approach is from the direct measurement of peak/plateau-width in lnT spectra. We find the statistical distribution of peak/plateau-width is quite abnormal and can be fitted very well by the summation of two Gaussian distribution functions, where the primary Gaussian center gives the typical width of resonant peaks of localized states while the secondary one gives exactly the typical width of the SNS plateaus. Excellent agreements are found between two approaches. The dependencies of SNS's properties on the scaling parameter L/ξ are also studied. We find the plateau-width of SNS almost does not depend on L/ξ while the peak-width of localized states decays exponentially with L/ξ, indicating the SNSs have more significant transmission contribution in longer systems. The essential quantities defined in this work also provide new ways for further quantitative study on statistical properties of the transmission of Anderson localized systems.

The rest of this paper is organized as follows. In section II, we introduce our model and basic properties of the lnT spectra. In section III, we present our numerical results and theoretical analysis of three correlation functions: the correlation of transmission C_T, the correlation of logarithmic transmission C_{lnT} and the field correlation C_t. We show that the half-width of C_{lnT} gives the typical plateau-width of SNS. The most-probable order of SNS can be obtained by comparing C_{lnT} and C_t. In section IV, we study statistical distributions of the peak/plateau-width (defined in the lnT spectra). The probability distributions are calculated from very large numbers of samples and can be fitted very well by the

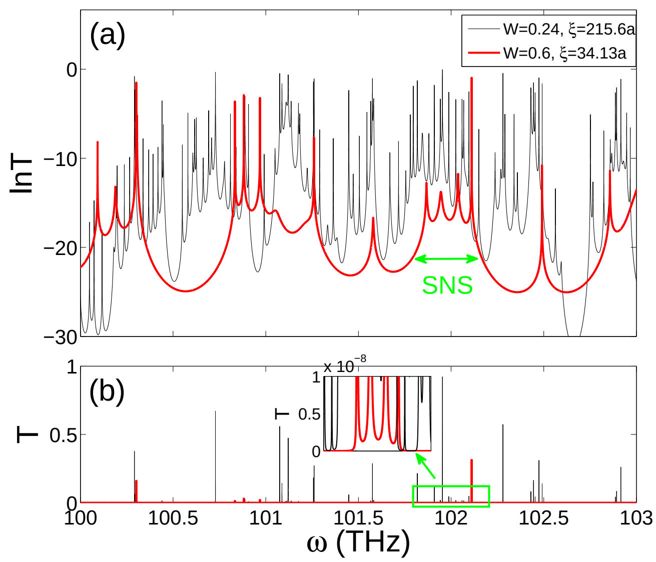

FIG. 1. (Color online) Typical logarithmic transmission spectra (a) and corresponding transmission spectra (b) of $L = 10\xi$ systems. Red bold line: $\xi = 34.13a$ ($W = 0.6$); black thin line: $\xi = 215.6a$ ($W = 0.24$). The SNS marked in (a) is enlarged in (b) for clear observation.

summation of two Gaussian distributions, where the first Gaussian center gives the typical width of resonant peaks of localized states and the second one gives the typical width of the SNS plateau. In section V, we discuss the system length dependency of the SNS. Finally, a summary of this work is given in section VI.

II. THE MODEL

We study 1D random stacks composed of binary-dielectric layers(called A and B) with thicknesses $d_A = d_B = 500nm$ and with refractive indices $n_A = 1.0$ and $n_B = 3.0 * (1 + W\gamma)$, where γ is a random number uniformly distributed in $[-0.5, 0.5]$ and W gives the randomness strength. Such a periodic on average model is an optical counterpart of the Anderson model of electronic systems and is widely used for localization studies[12,14,15,19,25]. The transmission coefficients of optical waves are calculated by standard transfer matrix method[25]. Without randomness, $W = 0$, the system exhibits a pass band in frequency range $(89.13, 124.1THz)$. Our study will focus on the range $(100.0, 103.2THz)$, where the localization length is almost a constant for a certain randomness $W(0.2 \le W \le 0.6)$[25]. The localization length is calculated by $\xi = -2L/\langle lnT\rangle$, where $\langle lnT\rangle$ is averaged from the 10^6 configurations satisfying $L \gg \xi$ ($L \approx 500\xi$).

Fig.1(a) shows two typical logarithmic transmission spectra with the same $L/\xi = 10$ but with different localization lengths, $\xi = 215.6a$(black thin) and $\xi =$

$34.14a$(red bold). We can see they have similar average height, being approximately $-2L/\xi = -20$. After averaged over a large number of configurations, the lnT spectrum becomes a flat line exactly falls on the mean value $\langle lnT \rangle = -20$. This is a natural result according to the single parameter scaling theorem that lnT follows the Gaussian distribution with mean value $-2L/\xi$. An interesting phenomenon in the lnT spectra is that there are some clear plateau-structures, such as the one marked by arrows in Fig.1(a). From the wave intensity distributions one can find those plateaus are formed by the SNSs(i.e., from the degenerate coupling between some spatially close localized states) embedded inside the configuration[26]. Since the peaks of localized states in the spectra are extremely sharp ($\sim e^{-L/\xi}$), the fluctuation of lnT is dominated by those plateaus. More importantly, these SNS may be essential for understanding the Anderson transition phenomenon[26]. After observed a large number of lnT spectra for fixed ξ and L/ξ, we find the plateaus appear with some intuitionistic regularities, (1) most plateaus have similar frequency width for a configuration; (2) the number of peaks on each plateau are similar. For example, for most plateaus of the $\xi = 34.14a$ system shown in Fig.1, the plateau width is about $0.3 \sim 0.4THz$ and the number of peaks are about $3 \sim 4$. In the following we will focus on the statistical properties of these SNSs and try to characterize them quantitatively.

To quantitatively study the SNS, we need to find proper physical quantities. Fig.1 shows that the plateaus of SNSs can hardly be distinguished from localized states in a T spectrum (see Fig.1(b)), but can be clearly identified by the plateaus in the lnT spectra (Fig.1(a)). So we try to define physical quantities in the lnT spectra for the SNS study. In the following, we will define and study the correlation function of lnT spectra in our first approach. Then, we will define the lnT peak/plateau width–Γ and study the statistical distribution of Γ in our second approach.

III. CORRELATION FUNCTIONS

According to the standard definition of mathematics, we define the correlation functions of T and lnT in frequency domain as:

$$C_T(\Delta\omega) = \frac{Cov(T_\omega, T_{\omega+\Delta\omega})}{\sigma(T_\omega)\sigma(T_{\omega+\Delta\omega})} \tag{1}$$

$$C_{lnT}(\Delta\omega) = \frac{Cov(lnT_\omega, lnT_{\omega+\Delta\omega})}{\sigma(lnT_\omega)\sigma(lnT_{\omega+\Delta\omega})} \tag{2}$$

where T is the transmission coefficient, σ is the standard deviation $\sigma(x) = \sqrt{\langle(x - \langle x\rangle)^2\rangle}$ and Cov is the covariance $Cov(x, y) = \langle x - \langle x\rangle\rangle \langle y - \langle y\rangle\rangle$. Physically C_T implicates the frequency correlation of transmitted wave intensity. Its Fourier transformation corresponds to the dynamical response at the outgoing interface, which have been intensively studied in weakly localized systems[27]. If two frequencies are on the same transmission peak, C_T is close to unity. Otherwise C_T will close to zero. Hence the half-width of C_T is usually used to characterize the linewidth of localized states[27]. Similarly, on the lnT spectra, since lnT of two frequencies on the same plateau are much larger than the mean value $\langle lnT\rangle$ and contribute a significantly large C_{lnT}. We expect C_{lnT} is close to unity when $\Delta\omega$ is smaller than a typical plateau-width and decays to zero when $\Delta\omega$ is larger than a typical plateau-width.

Meanwhile, we also study the field correlation function,

$$C_t(\Delta\omega) = \frac{< t_\omega t_{\omega+\Delta\omega}{}^* + t_\omega{}^* t_{\omega+\Delta\omega} >}{< T_\omega > + < T_{\omega+\Delta\omega} >} \tag{3}$$

where t is complex transmission coefficient and $T = tt^*$. The complex amplitude of incident wave is chosen to be $E_{in}(x = 0) = 1$ so that at the outgoing interface $t = E(x = L) = |E|e^{i\phi}$, where E is the complex electronic field and ϕ is its phase. Similar to C_T, the physical implication of C_t is simply the frequency correlation of transmitted field, i.e., the field correlation(C_T is the intensity correlation). The definition of C_t is the same as that in ref.[7], which could be negative valued, depending on the averaged phase difference $\Delta\phi$ between ω and $\omega + \Delta\omega$. Generally, when $\Delta\omega$ crosses a resonant peak, the phase of t jumps π[7,25]. Hence the phase difference between $t(\omega)$ and $t(\omega + \Delta\omega)$ is approximately $\Delta\phi = \phi(\omega) - \phi(\omega+\Delta\omega) \approx \pi$. Then $t_\omega t^*_{\omega+\Delta\omega} + t^*_\omega t_{\omega+\Delta\omega} = |E_\omega||E_{\omega+\Delta\omega}| \cdot 2cos(\Delta\phi)$ becomes negative and so that C_t is negative. When $\Delta\omega$ gets to the value which typically contains two peaks (plus one average distance between two peaks), C_t will be positive since $\Delta\phi \approx 2\pi$, and so forth. Hence when increasing $\Delta\omega$, we expect $C_t(\Delta\omega)$ to oscillate between positive and negative. Each time $C_t(\Delta\omega)$ changes its sign, the average number of resonant peaks in $\Delta\omega$ increases one. So $C_t(\Delta\omega)$ provides a way to check the average resonant peak number in a certain frequency range.

In our calculations we set the original frequency point $\omega = 100THz$ without lose of generality. The correlation functions calculated from a very large number (2×10^7) of configurations for different ξ are shown in Fig.2(a), where the ratio L/ξ is fixed at 10. The solid, dashed and dotted curves respectively represents C_T, C_t and C_{lnT}. The red curves marked by crosses correspond to the smaller ξ system and the black curves marked by solid dots correspond to the larger ξ system. In Fig.2(b) the transverse axis is shown in logarithmic scale.

Let us first discuss C_T. Fig.2(a) shows C_T is a very singular function at the origin $\Delta\omega \to 0$. The linewidth $\Delta\omega$ of localized states, corresponding to $C_T(\Delta\omega) = 0.5$, is larger in the smaller ξ systems. This is in consistent with the observation in Fig.1 that the linewidth of resonant peaks in smaller ξ system is larger. Detailed data gives that the halfwidth of the larger ξ system (black solid curve) is about $0.382GHz$ and the smaller ξ system (red

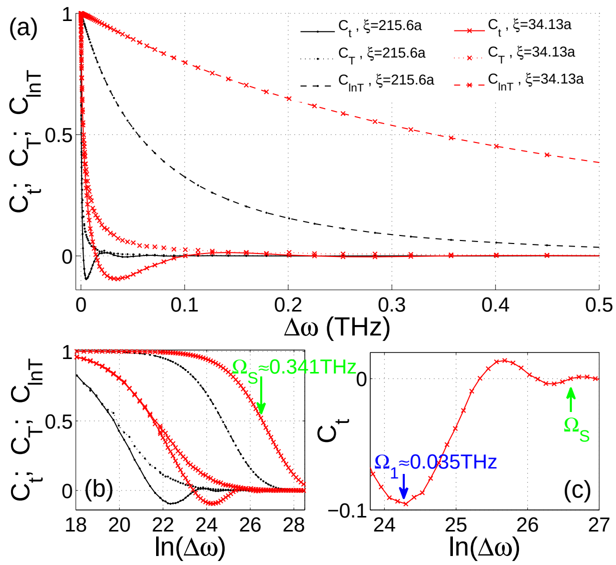

FIG. 2. (Color online) (a): Correlation functions of $L = 10\xi$ systems calculated from 2×10^7 configurations. Red: $\xi = 34.13a$; Black: $\xi = 215.6a$. For each color, the correlation functions from the top down are respectively C_{lnT}, C_T and C_t. (b): Same as (a) but with $\Delta\omega$ shown in logarithmic scale. (c): Detailed view of the oscillations of $C_t(\xi = 34.13a)$. The half-width of C_{lnT}, Ω_S, gives the typical plateau-width of SNS. The first minimum of C_t, Ω_1, gives the typical width of a resonant peak.

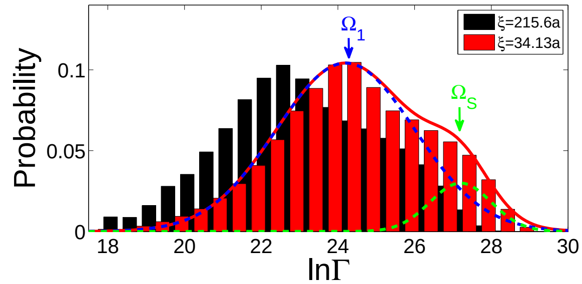

FIG. 3. (Color online) Probability distribution of the peak/plateau-width Γ(see the text for definition) measured from logarithmic transmission spectrum. The distribution function can be fitted very well by the summation(red solid curve) of two Gaussian functions(dashed curves). The primary(left, blue) Gaussian center corresponds to the typical width of resonant peaks of localized states. The secondary(right, green) Gaussian center corresponds to the typical plateau-width of SNS.

solid curve) is $2.483GHz$. This is also quantitatively in consistent with the observation from the lnT spectra.

C_{lnT} is a much smoother function than C_T in the $\Delta\omega \to 0$ limit. Take the $\xi = 34.13a$ system (red curves) for example. C_T rapidly falls to zero near $ln(\Delta\omega) \approx 24$. C_{lnT} exhibits a plateau at the origin, then drops to 0.5 at $ln(\Delta\omega) \approx 26.6$, i.e., its half-width is about $0.341THz$, being much larger than that of C_T. Such contrast reflects the different geometry properties between the T and lnT spectrum. On the T spectrum, C_T falls to zero typically when two frequencies are not on the same peak. Since the peaks of localized states are exponentially sharp, C_T decreases rapidly as increasing $\Delta\omega$. However, on the lnT spectrum, there exist many plateaus formed by SNSs, as shown in Fig.1(a). Those plateaus are usually higher than the average transmission background $\langle lnT\rangle$. When two frequencies are on the same plateau but not the same peak, it will still contribute a significantly large value to C_{lnT}. Hence *the halfwidth of* C_{lnT}*, which is denoted as* Ω_S *in this paper, approximately gives the typical plateau-width of SNS in* lnT *spectrum.*

With the help of C_t, we can find more detailed information of SNS, such as the average SNS order, which is the average number of coupled localized states in one SNS. Similar to C_T, C_t also shows sharp singularity at the origin and falls quickly as increasing $\Delta\omega$. Interestingly, C_t shows some oscillations around $C_t = 0$ in the region where C_T falls to nearly zero, see Fig.2(c). As discussed earlier in this section, those oscillations can be understood from the π-phase jumps of resonant peaks of the localized states. The different order minimal/maximal points of the C_t correspond to the frequency ranges which can accommodate certain number of resonant peaks. We denote the $\Delta\omega$ at the first minimum of C_t as Ω_1, which is the the typical frequency width of a resonant peak. We also denote nth order minimal/maximal points Ω_n as the average frequency range which can accommodate n resonant peaks. So C_t is the ruler of the number of resonant peaks in a frequency range. With Ω_n in our mind, we can see that the mean width of SNS plateaus, obtained by C_{lnT}, can accommodate about $3 \sim 4$ resonant peaks, as indicated by the green arrows in the Fig.2(c). Actually, this average number of resonant peaks in a SNS agrees very well with our direct observation of many spectra.

IV. PEAK/PLATEAU-WIDTH STATISTICS

To make a test and verify of the characters of SNS obtained from C_{lnT} and C_t, we next try our second approach, *i.e.* direct measure of the peaks/plateaus-width in a large number of lnT spectra. Since the ensemble average of lnT spectra–$\langle lnT\rangle$ is well defined, it is natural to *define the peaks/plateaus-width* Γ *as the frequency interval where* lnT *is always higher than* $\langle lnT\rangle$. From direct observation of the lnT spectra, one can find that $\langle lnT\rangle$ crosses lots of sharp peaks of localized states and fewer SNS plateaus. More precise statistical results should be obtained from a large number of realizations. We find that the statistical distribution of Γ is extremely skewed. The reason is that in Anderson localized systems the peak-width of localized states is exponentially small and the width of coupled-peaks also scales exponen-

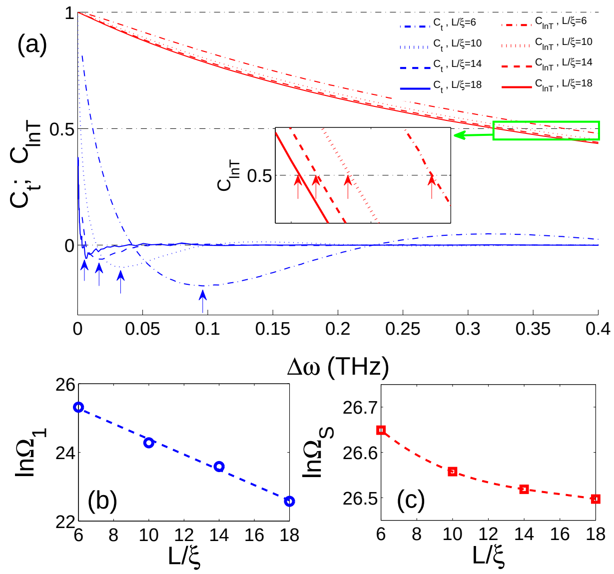

FIG. 4. (Color online) (a): C_t (lower four blue curves) and C_{lnT} (upper four red curves) for different L/ξ systems. For each set of four curves, $L/\xi = 6, 10, 14, 18$ from left to right. The inset gives detailed view at the halfwidth of C_{lnT}. (b): The typical peak-width of localized states Ω_1 (marked by blue arrows in (a)) as a function of L/ξ. (c): The typical plateau-width of SNS Ω_S (marked by red arrows in (a)) as a function of L/ξ. Ω_1 decays exponentially as increasing L/ξ while Ω_S is insensitive to L/ξ.

tially with the system length[5,7,25]. This is very similar to the probability distribution of the dimensionless conductance g, which is extremely skewed (nearly log-normal). Early study[4] on Anderson localization has shown that it is better to study the additive quantity–lng, which is nearly Gaussian distributed. Similarly, instead of Γ, we will study the $ln\Gamma$ distribution, which is likely Gaussian distributed.

We have measured Γ in 10^5 spectra with high frequency precision that can distinguish each resonant peak in the $L = 10\xi$ systems[28]. The probability distributions of $ln\Gamma$ for two different ξ systems are shown in Fig.3. Take the $\xi = 34.13a$ system as the example. (All the following discussions also apply to the $\xi = 215.6a$ system.) The probability distribution is roughly Gaussian-like, where a clear maximum is found at $ln\Gamma \approx 24.3(\Gamma \approx 0.035THz)$. This is exactly the $ln(\Omega_1)$ at first minimum of C_t shown in Fig.2. As shown before, this value corresponds to the typical frequency range occupied by one single resonant peak. Such a result is coincident with our direct observation of the lnT spectra that the most probable peaks/plateaus crossed by $\langle lnT\rangle$ are those single peaks. Suppose the spectrum contains only sharp peaks but none plateaus. $ln(\Delta\omega)$ should be nearly Gaussian distributed with a single maximum at the mean value. However, the real probability distribution of $ln\Gamma$ shows a strange shoulder at $ln\Gamma \approx 26.6(\Gamma \approx 0.341THz)$. Comparing with Fig.2 we find it exactly corresponds to the halfwidth of C_{lnT}, Ω_S, which is just the typical width of SNS plateau, referring to the physical meaning of C_{lnT}. Actually, the measured probability distribution can be fitted very well by the summation of two Gaussian distributions, as shown by the curves in Fig.3. The primary(left) Gaussian center gives the width of single resonant peaks while the secondary(right) one gives exactly the typical width of the SNS plateau. Such a strange probability distribution implies substantial SNS plateaus with similar frequency widths contribute significantly to the probability distribution function of $ln\Gamma$, resulting the characteristic shoulder of SNS.

It is first time to see directly from the statistical distribution that the SNS is clearly distinguished from other localized states. From the distribution, we can see that the logarithmic plateau-width of SNS is Gaussian distributed and its mean value agrees excellently with the results of correlation functions in third section, as expected. So, both C_{lnT} and the probability distribution of $ln\Gamma$ provides proper physical values for characterizing the SNS.

V. SYSTEM LENGTH DEPENDENCIES

Both the correlation functions and the probability distribution of $ln\Gamma$ suggest that the SNS favors a specific frequency width and a specific number(order) of localized states. But those studies are done with certain L/ξ. Next, to drive this conclusion further, we will calculate the correlation functions in the systems with different L/ξ. Fig.4(a) shows C_t (the lower four blue curves) and C_{lnT} (the upper four red curves) functions for several L/ξ values, where ξ is fixed at $34.13a$ ($W = 0.6$) and $L/\xi = 6, 10, 14, 18$ from left to right. It clearly shows the peak width Ω_1 of resonant peaks of localized states (marked by blue arrows) decreases dramatically as increasing L while the decrease of plateau-width Ω_S(halfwidth of C_{lnT} marked by red arrows) is much smaller. Ω_1 and Ω_S versus L/ξ are shown in Fig.4(b) and (c). We can see Ω_1 decays exponentially with L ($\sim e^{-L/\xi}$), as expected from traditional understanding[5,6,9]. However, the decrease of Ω_S is almost negligible and obviously much slower than Ω_1. Actually, the plateau-width of SNS is determined by the intrinsic coupling strength(repulsing distance) between localized states, which almost does not depend on L[26]. Such a distinction naturally results a picture that, as increasing L more and more, the peaks of localized states become so sharp that they are almost undetectable, but the SNS plateaus are still there. In our numerical statistics, the number(order) of localized states increases nearly linearly with L/ξ. In Ref.[26] it is further argued that those SNSs (called intrinsic short necklace states) have main contribution to the fluctuation of transmission and also affect the value of localization length.

VI. SUMMARY

In summary, we have defined the basic quantities for SNS study and investigate the statistical properties of SNS in the strongly localized systems. We find two approaches to quantitatively study SNS properties. The first approach is based on the correlation functions and the second one is based on direct measurements of the peak/plateau-width in logarithmic transmission spectra. In the first approach, we defined the correlation function C_{lnT} in lnT spectra and show that the typical width of SNS plateaus can be characterized by the half-width of C_{lnT}. And, with the help of C_t (correlation function of transmission coefficient t), the most-probable order of SNS can be obtained. In the second approach, we defined the peak/plateau-width Γ in lnT spectra and studied the probability distribution of $ln\Gamma$ directly measured from a large number of spectra. The probability distribution of $ln\Gamma$ shows a novel shoulder and it can be fitted very well by the summation of two Gaussian distributions. We showed that the first one is from the distribution of localized state peaks and the second one is from the plateaus of SNS. The center of the second distribution gives the average frequency width of SNS plateaus, which agrees very well with the value obtained from correlation function. As increasing the system length L, the plateau-width of SNS decays very slowly, compared with the exponentially-decayed peak-width of localized states. Finally, we note that the methods we used in our paper is not limited to the transport properties of localized system. They may be used for studying other statistical quantities which have similar properties with the transmission of localized systems.

This work is supported by the NSFC (Grant Nos. 11004212, 11174309, and 60938004), and the STCSM (Grant Nos. 11ZR1443800 and 11JC1414500).

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[28] Note that each spectrum covers a wide frequency range, where a lot of peaks and plateaus will be cutted by $\langle lnT \rangle$. Such that the ensemble of 10^5 spectra is considerably large, with about 10^7 sample peaks/plateaus.